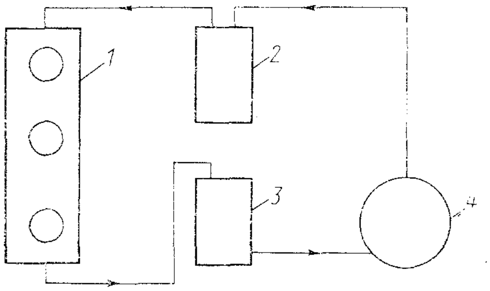

Fig. 2

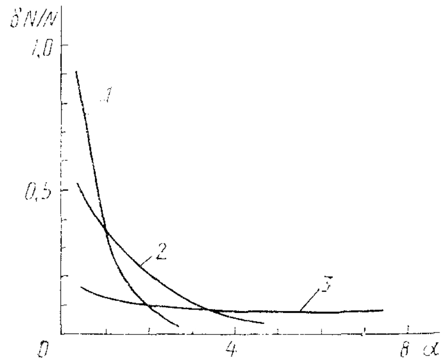

Fig. 3

Fig. 2. Monodispersed droplet generator.

Fig. 3. Relative quantity of droplets $\delta N/N$ vs angle of deviation α, $sr \cdot 10^4$; chamber pressure: 1) 419 torr; 2) 711; 3) 740.

experiment. Figure 3 shows the dependence of the relative quantity of droplets $\delta N/N$ vs deviation angle α. For lower pressures in the vacuum chamber these values are not shown, since in this case the deviation did not exceed the experimental error.

NOTATION

N, quantity of particles studied; δN, number of particles with identical deviation angle; α, angular deviation.

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STABILITY OF PERIODIC FLOW IN A MICROPOLAR FLUID

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The stability of unidirectional periodic flow in a micropolar fluid is treated. An analytic expression is found for the critical Reynolds number of stability loss.

Among the non-Newtonian fluids, for whose description one needs additional hydrodynamic variables, one of the best known is the so-called micropolar fluid. Along with the classical hydrodynamic variables (velocity, pressure, and so on), for macroscopic description of this medium one requires three additional variables, interpreted as the components of the angular velocity of microrotation Ω.

The equations of a micropolar fluid are substantially more complicated than the Navier–Stokes equations. This renders the construction of rigorous solutions, and particularly the stability analysis, more difficult. Most of the stability studies of a micropolar fluid were carried out numerically (see, for example, the stability studies of Couette [1], Taylor–Couette [2], and Benard–Rayleigh [3] flows). It is, therefore, of interest to investigate a problem for which one can expect to obtain analytic results in the study of linear and weakly linear stability.

As will be seen in what follows, such an example is unidirectional periodic flow of a micropolar fluid, induced by an external force $F \sim \sin y$, directed along the x-axis (the so-called

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Kolmogorv flow). The stability of Kolmogorov flow in a viscous incompressible fluid was investigated analytically in [4-6] within the Navier–Stokes equations. Reviews of theoretical, numerical, and experimental results are contained in [7, 8].

Statement of the Problem. Introduction of Slow Variables. Consider planar flow of a micropolar fluid, induced by a massive force F periodic in one of the coordinates. The equations of motion are conveniently represented in the form:

$$\rho\left[\frac{\partial}{\partial t}(\Delta\psi)+\frac{\partial(\Delta\psi,\ \psi)}{\partial(x,\ y)}\right]=(\mu+k)\Delta^2\psi+k\Delta\psi-F(y), \tag{1}$$

$$\rho J\left[\frac{\partial\sigma}{\partial t}+\frac{\partial(\sigma\psi)}{\partial(x,\ y)}\right]=\gamma\Delta\sigma-k(2\sigma+\Delta\psi). \tag{2}$$

For planar flow the angular velocity of microrotation has a single nontrivial component, denoted by σ: $\Omega = (0,\ 0,\ \sigma)$.

We convert now all variables to dimensionless form. The unperturbed Kolmogorov flow $\psi = Ua\cos(y/a)$ is used to select the units of length and velocity, i.e., the velocity is measured in units of U, and the length – in units of a. We use ρ for unit density. As a result Eqs. (1), (2) acquire the form:

$$\frac{\partial}{\partial t}\Delta\psi+\frac{\partial(\Delta\psi,\ \psi)}{\partial(x,\ y)}=\frac{1}{\mathrm{R}}\left[\Delta^2\psi+\lambda\Delta\sigma-\left(1-\frac{\lambda^2}{l^2+2\lambda}\right)\cos y\right], \tag{3}$$

$$j\left[\frac{\partial}{\partial t}\sigma+\frac{\partial(\sigma,\ \psi)}{\partial(x,\ y)}\right]=\frac{1}{\mathrm{R}}\left[l^2\Delta\sigma-\lambda(2\sigma+\Delta\psi)\right]. \tag{4}$$

Standard notations have been used in Eqs. (1)-(4) for the Laplacian and Jacobian. Besides, the dimensionless parameters R, λ, ℓ, j, defined by the relations

$$\mathrm{R}=\frac{\rho Ua}{\mu+k},\quad \lambda=\frac{k}{\mu+k},\quad l=\sqrt{\frac{\gamma}{(\mu+k)a^2}},\quad j=\frac{J}{a^2}, \tag{5}$$

were introduced in Eqs. (3) and (4). The physical meaning of these parameters is obvious: R is the Reynolds number, λ is a measure of particle coupling with its surrounding, for $\lambda = 0$ Eq. (3) transforms to the Navier–Stokes equations, ℓ is a dimensionless "internal" length in a micropolar fluid, calculated from the kinetic coefficients, and j is the square of another "internal" length, calculated from the microment of inertia.

It is assumed that the Kolmogorov flow $\psi=\cos y$, $\sigma=\frac{\lambda}{l^2+2\lambda}\cos y$ loses stability at some critical Reynolds number R_*. In the case of low supercriticality, i.e., when R differs little from the critical number R_*, it is convenient to introduce a small parameter ε by the rule

$$\mathrm{R}^{-1}=\mathrm{R}_*^{-1}(1-\varepsilon^2) \tag{6}$$

with the following deformation of space–time coordinates

$$T=\varepsilon^4 t,\quad X=\varepsilon x,\quad Y=y. \tag{7}$$

Arguments for the usefulness of such choice of scales were first formulated in [9-11], as applied to the study of stability of convective flows. As shown by Shivashinsky [6], for Kolmogorov flow of a Newtonian fluid the use of scale transformations (6), (7) makes it possible to obtain relatively simply the basic results of linear stability theory [4, 5], along with new results in weakly linear stability.

In the new variables the equations of motion (3), (4) are

$$\varepsilon^4\frac{\partial}{\partial t}(\psi_{yy}+\varepsilon^2\psi_{xx})+\varepsilon\frac{\partial(\psi_{yy},\ \psi)}{\partial(x,\ y)}+\varepsilon^3\frac{\partial(\psi_{xx},\ \psi)}{\partial(x,\ y)}=$$

$$=\frac{1-\varepsilon^2}{\mathrm{R}_*}\left[\psi_{yyyy}+2\varepsilon^2\psi_{xxyy}+\varepsilon^4\psi\psi_{xxxx}+\lambda(\sigma_{yy}+\varepsilon^2\sigma_{xx})-\left(1-\frac{\lambda^2}{l^2+2\lambda}\right)\cos y\right], \tag{8}$$

$$j\left[\varepsilon^4\frac{\partial\sigma}{\partial t}+\varepsilon\frac{\partial(\sigma,\ \psi)}{\partial(x,\ y)}\right]=\frac{1-\varepsilon^2}{\mathrm{R}_*}\left[l^2(\sigma_{yy}+\varepsilon^2\sigma_{xx})-\lambda(2\sigma+\psi_{yy}+\varepsilon^2\psi_{xx})\right]. \tag{9}$$

In Eqs. (8), (9) and everywhere in the following we use the new variables T, X, Y, which are denoted for convenience by the lower-case letters t, x, y, respectively.

Since the perturbed flow is naturally assumed to be periodic, we integrate (8) and (9) over a period. As a result we reach the integral relations:

$$\varepsilon^4 \frac{\partial}{\partial t} \int_0^{2\pi} \psi_{xx} dy + \varepsilon \frac{\partial}{\partial x} \int_0^{2\pi} \psi_y \psi_{xx} dy = \varepsilon^2 \frac{1-\varepsilon^2}{R_*} \frac{\partial^4}{\partial x^4} \int_0^{2\pi} \psi dy + \lambda \frac{1-\varepsilon^2}{R_*} \frac{\partial^2}{\partial x^2} \int_0^{2\pi} \sigma dy, \tag{10a}$$

$$j\left[\varepsilon^4 \frac{\partial}{\partial t} \int_0^{2\pi} \sigma dy + \varepsilon \frac{\partial}{\partial x} \int_0^{2\pi} \psi_y \sigma dy\right] = \frac{1-\varepsilon^2}{R_*}\left[\varepsilon^2 \int_0^{2\pi} (l^2\sigma_{xx} - \lambda\psi_{xx})\, dy - 2\lambda \int_0^{2\pi} \sigma dy\right], \tag{10b}$$

which are used in the following to calculate the critical number R_*.

Determination of the Critical Reynolds Number. The solution of the problem is sought in the form of asymptotic series in a small parameter:

$$\psi = \psi_0 + \varepsilon\psi_1 + \varepsilon^2\psi_2 + \ldots, \quad \sigma = \sigma_0 + \varepsilon\sigma_1 + \varepsilon^2\sigma_2 + \ldots \tag{11}$$

Substituting (11) into (8), (9), we find in the vanishing approximation:

$$\frac{\partial^4\psi_0}{\partial y^4} + \lambda \frac{\partial^2\sigma_0}{\partial y^2} = \left(1 - \frac{\lambda^2}{l^2+2\lambda}\right)\cos y, \tag{12a}$$

$$l^2 \frac{\partial^2\sigma_0}{\partial y^2} = \lambda\left(2\sigma_0 + \frac{\partial^2\psi_0}{\partial y^2}\right). \tag{12b}$$

Integrating (12a), (12b), and taking into account that we seek a periodic solution, we obtain

$$\psi_0 = \cos y + \Phi_0(x, t), \tag{13a}$$

$$\sigma_0 = \frac{\lambda}{l^2+2\lambda}\cos y. \tag{13b}$$

The solvability condition of (10a), (10b) are automatically satisfied by the solution (13).

In the first approximation we find from (8), (9) with account of (13)

$$\frac{\partial^4\psi_1}{\partial y^4} + \lambda \frac{\partial^2\sigma_1}{\partial y^2} = -R_* \frac{\partial\Phi_0}{\partial x}\sin y, \tag{14a}$$

$$l^2 \frac{\partial^2\sigma_1}{\partial y^2} - \lambda\left(2\sigma_1 + \frac{\partial^2\psi_1}{\partial y^2}\right) = R_* j \frac{\lambda}{l^2+2\lambda} \frac{\partial\Phi_0}{\partial x}\sin y. \tag{14b}$$

Analysis of (14a), (14b) shows that the solution must be sought in the form:

$$\psi_1 = -R_* A_1 \frac{\partial\Phi_0}{\partial x}\sin y + \Phi_1(x, t), \tag{15a}$$

$$\sigma_1 = -\lambda R_* a_1 \frac{\partial\Phi_0}{\partial x}\sin y. \tag{15b}$$

Substitution of (15) into (14) leads to the following constant values:

$$A_1 = 1 + \lambda^2 a_1, \quad a_1 = \frac{l^2+2\lambda+j}{(l^2+2\lambda)(l^2+2\lambda-\lambda^2)}. \tag{16}$$

The solvability condition of (10a) acquires the following form in the first approximation

$$\frac{\partial}{\partial x}\left(\int_0^{2\pi} \frac{\partial\psi_0}{\partial y} \frac{\partial^2\psi_0}{\partial x^2} dy\right) = \frac{\lambda}{R_*} \frac{\partial^2}{\partial x^2} \int_0^{2\pi} \sigma_1 dy. \tag{17}$$

Both terms in (17) vanish identically due to (13a) and (15b), respectively. The second solvability condition

$$j \frac{\partial}{\partial x} \int_0^{2\pi} \sigma_0 \frac{\partial\psi_0}{\partial y} dy + \frac{2\lambda}{R_*} \int_0^{2\pi} \sigma_1 dy = 0 \tag{18}$$

is also satisfied automatically, since both integrals in (18) vanish identically at the solution (13), (15b).

In the second approximation we obtain from (8), (9), with account of (13), (15):

$$\frac{\partial^4\psi_2}{\partial y^4}+\lambda\frac{\partial^2\sigma_2}{\partial y^2}=-R_*\frac{\partial\Phi_1}{\partial x}\sin y-R_*^2A_1\left(\frac{\partial\Phi_0}{\partial x}\right)^2\cos y, \tag{19a}$$

$$l^2\frac{\partial^2\sigma_2}{\partial y^2}-\lambda\left(2\sigma_2+\frac{\partial^2\psi_2}{\partial y^2}+\frac{\partial^2\Phi_0}{\partial x^2}\right)=\lambda R_* j\left[\frac{1}{l^2+2\lambda}\frac{\partial\Phi_1}{\partial x}\sin y+\right.$$

$$\left.+R_*a_1\left(\frac{\partial\Phi_0}{\partial x}\right)^2\cos y+R_*\left(a_1-\frac{A_1}{l^2+2\lambda}\right)\frac{\partial^2\Phi_0}{\partial x^2}\sin^2 y\right]. \tag{19b}$$

It is seen from (19) that the periodic solution must be sought in the form:

$$\psi_2=-R_*A_1\frac{\partial\Phi_1}{\partial x}\sin y-R_*^2A_2\left(\frac{\partial\Phi_0}{\partial x}\right)^2\cos y-R_*^2B_2\frac{\partial^2\Phi_0}{\partial x^2}\cos 2y+\Phi_2(x,\ t), \tag{20a}$$

$$\sigma_2=-\lambda\left[R_*a_1\frac{\partial\Phi_1}{\partial x}\sin y+R_*^2a_2\left(\frac{\partial\Phi_0}{\partial x}\right)\cos y+R_*^2b_2\frac{\partial^2\Phi_0}{\partial x^2}\cos 2y\right]+c_2\frac{\partial^2\Phi_0}{\partial x^2}. \tag{20b}$$

Substituting (20) into (19), we arrive at a system of simple linear equations in the unknowns A_2, a_2, B_2, b_2, c_2. Solving this system, we finally find:

$$A_2=A_1+\lambda^2a_2,\quad a_2=(A_1+ja_1)(l^2+2\lambda-\lambda^2)^{-1}, \tag{21a}$$

$$B_2=\lambda^2b_2/4,\quad b_2=\frac{1}{2}j\left(\frac{A_1}{l^2+2\lambda}-a_1\right)(4l^2+2\lambda-\lambda^2)^{-1}, \tag{21b}$$

$$c_2=-\frac{1}{2}+\frac{1}{4}jR_*^2\left(\frac{A_1}{l^2+2\lambda}-a_1\right). \tag{21c}$$

The solvability condition (10a) is in the second approximation

$$R_*\frac{\partial}{\partial x}\int_0^{2\pi}\left(\frac{\partial\psi_1}{\partial y}\frac{\partial^2\psi_0}{\partial x^2}+\frac{\partial\psi_0}{\partial x^2}\frac{\partial^2\psi_1}{\partial x^2}\right)dy=\frac{\partial^4}{\partial x^4}\int_0^{2\pi}\psi_0dy+\lambda\frac{\partial^2}{\partial x^2}\int_0^{2\pi}(\sigma_2-\sigma_0)\,dy$$

and, following elementary transformations, leads to the required expression for the critical Reynolds number of stability loss:

$$R_*=\sqrt{\frac{2-\lambda}{A_1-\frac{1}{2}j\lambda\left(\frac{A_1}{l^2+2\lambda}-a_1\right)}}. \tag{22}$$

The second solvability condition of (10b)

$$R_*j\frac{\partial}{\partial x}\int_0^{2\pi}\left(\sigma_0\frac{\partial\psi_1}{\partial y}+\sigma_1\frac{\partial\psi_0}{\partial y}\right)dy=\frac{\partial^2}{\partial x^2}\int_0^{2\pi}(l^2\sigma_0-\lambda\psi_0)\,dy-2\lambda\int_0^{2\pi}(\sigma_2-\sigma_0)\,dy$$

is transformed into an identity following substitution of the results obtained (13), (15), (20b), (21b), (21c).

Thus, the critical number R_* is determined by Eq. (22) and depends on the three dimensionless similarity parameters λ, ℓ, j. The equation obtained is, as well as we know, a unique analytic result in stability theory of a macropolar fluid. In the case of a Newtonian fluid $\lambda = 0$, and Eq. (22) transforms to the well-known Meshalkin–Sinai equation $R_* = \sqrt{2}$ [4].

Analysis of (22) leads to the following conclusion: with increasing $\lambda = k/(\mu + k)$ from 0 to 1 and fixed values of the remaining similarity parameters ℓ and j there is a monotonic decrease of R_*, i.e., the flow stability is reduced. This result is not in agreement with results of other authors (see, for example, [2]), observing stability enhancement of a micropolar fluid in comparison with a Newtonian one. This discrepancy is basically an artefact of our determination of the Reynolds number (5) from the kinetic coefficient $\mu + k$. Using the more customary definition $Re = \rho Ua/\mu$ [in this case the equations of motion (3), (4) and all subsequently obtained results are more awkward, therefore it seemed advisable to restore the definition (5)], then with account of the relation $Re = R(1 - \lambda)^{-1}$ the behavior of $Re_* = Re_*(\lambda)$ with increasing λ and fixed ℓ and j values is almost opposite for all ℓ and j values. This is not directly seen from expression (22), but in real situations, when both internal lengths ℓ and $\sqrt{j}$ are very small, (22) transforms to the asymptotic equation

$$\mathrm{Re}_* = \sqrt{2}\,\frac{1-\dfrac{\lambda}{2}}{1-\lambda}\left[1+\frac{1}{4}\left(\frac{l^2}{2-\lambda}-\frac{j}{2}\right)+O(l^4,\ l^2 j,\ j^2)\right],$$

which expresses smoothly the fact of stability enhancement with increasing "extent of micropolarity" λ.

We note that for some values of the parameters ℓ and j the dependence $Re_* = Re_*(\lambda)$ becomes nonmonotonic, and the flow stability of the microplar fluid can be reduced. In the region of small internal lengths $\ell \ll 1$, $j \ll 1$ this is possible when $j \gg \ell^2$. In fact, when the extent of micropolarity is of order j, with account of (16) we obtain from (22)

$$\mathrm{Re}_* \approx \sqrt{2}\left(1+\frac{3}{2}\lambda\right)^{1/2}\left(1+\frac{j+2\lambda}{4}+\frac{j^2}{8\lambda}\right)^{-1/2},$$

whence it is seen that in the region $0 < \lambda < j/2$ there indeed occurs a stability reduction, $Re_* < \sqrt{2}$. We note, however, two facts. Firstly, the maximum Re_* reduction in comparison with $\sqrt{2}$ is a small quantity O(j). Secondly, the internal length $\sqrt{j}$ is of the order of a typical size of a microstructure in the micropolar fluid, while at the same time the other internal length ℓ, calculated from the kinetic coefficients, is usually substantially larger than $\sqrt{j}$. In many studies, starting with the well known [12], the convective terms in the equation for the angular velocity of microrotation have been generally discarded, i.e., it was assumed that $j = 0$. Detailed analysis of (22) for $\ell \geqslant \sqrt{j}$ shows that in this region the behavior of $Re_*(\lambda)$ is standard, and stability is enhanced with increasing λ. On the whole, the nonmonotonic behavior of micropolar fluid characteristics reflects the complex internal structure of this medium. In other problems complex effects were observed in a recent study [13].

We now discuss briefly the general scheme of further consideration. The procedure described of constructing asymptotic expansions can be extended further. In the third approximation we again obtain relation (22) as an asymptotic solvability condition from the integral relations (10). Therefore, to obtain really new results it is necessary to turn to the following fourth approximation. Following highly awkward calculations, as an asymptotic solvability condition one obtains the following evolution equation in the function $\Phi_0(x, t)$:

$$\frac{\partial\Phi_0}{\partial t}+A\,\frac{\partial^2\Phi_0}{\partial x^2}+B\,\frac{\partial^4\Phi_0}{\partial x^4}+C\left(\frac{\partial\Phi_0}{\partial x}\right)^2\frac{\partial^2\Phi_0}{\partial x^2}=0, \tag{23}$$

where A, B, C are expressed in terms of the similarity parameters λ, ℓ, j in terms of equations not provided here. An equation of type (23) was encountered earlier in problems of Benard convection in an almost isolated fluid layer [9-11].

Thus, for $R > R_*$ Kolmogorov flow loses its stability. In the case of low supercriticality, $R - R_* \ll 1$, a longitudinal vortex structure is generated with characteristic sizes $x \sim (R - R_*)^{-1/2}$, $y \sim 1$ and characteristic evolution time $t \sim (R - R_*)^{-2}$ [all this follows directly from (6), (7)]. Within the main approximation this large-scale vortex structure has the form (13), and it evolves according to Eq. (23).

We now discuss the issue of possible experimental realization of the flow considered. In the case of a Newtonian fluid such an experiment was undertaken in [14]: a thin electrolytic layer was placed in an external constant magnetic field with a periodic vertical component H_z, so that the role of the external force was played by the Lorentz force $[H \times j]$. Strictly speaking, for the theoretical description it is necessary to take into account the fluid interaction with the container (such as adhesion to the walls), as well as the three-dimensionality of the flow realized. Nevertheless, the authors of [14] preferred to assume that the flow was primarily two-dimensional, not accounting for the true boundary conditions, but modifying the equations themselves by supplementing the Navier-Stokes equations by terms proportional to the velocity components u and v, respectively, and assigning a number to account for friction with the bottom of the vessel. It is particularly simple to smooth out the modified Navier–Stokes equation in the form of a Helmholtz equation [i.e., in the form (1) with $k = 0$]: the term $\Delta^2\psi$ in the right-hand side must be replaced by $\Delta^2\psi - a\Delta\psi$.

We assume that this hypothesis is also valid in the case of a micropolar fluid. In this case all our considerations, starting with (7), remain valid if the friction coefficient with the container walls a is a fourth order quantity $a = \alpha\varepsilon^4$, $\alpha = O(1)$. Equation (8) is slightly modified in this case [the term $\alpha\varepsilon^4(\psi_{yy} + \varepsilon^2\psi_{xx})$ appears inside the square brackets in the right-hand side of (8)]. The solvability condition (10) is also modified somewhat – the right-hand side is supplemented by the term

$$-\alpha\varepsilon^4\frac{1-\varepsilon^2}{R_*}\int_0^{2\pi}\psi_{xx}dy.$$

Obviously, all results up to the third approximation, inclusive, are retained. In particular, R_* is primarily described by relation (22). Though the fourth approximation is changed with account of friction, from the solvability condition it can be concluded that a nontrivial new contribution is provided only by the additional term noted above. As a result we obtain, instead of (23), the following modified evolution equation

$$\frac{\partial\Phi_0}{\partial t}+A\frac{\partial^2\Phi_0}{\partial x^2}+B\frac{\partial^4\Phi_0}{\partial x^4}+C\left(\frac{\partial\Phi_0}{\partial x}\right)^2\frac{\partial^2\Phi_0}{\partial x^2}-\frac{\alpha}{R_*}\Phi_0=0 \tag{24}$$

with the former constants A, B, C.

Adding the new term to (23) leads to a qualitative change in the properties of the solution $\Phi_0(x, t)$. In fact, we state the problem of possible polynomial integral equations (23), (24). A useful criterion of existence of such integrals, recently obtained in [15], shows that for Eq. (23) there exists a unique polynomial integral

$$I=\int_{-\infty}^{\infty}\Phi_0(x,\ t)\,dx,$$

while for Eq. (24) there exist no such integrals for $\alpha \neq 0$.

Stability of Arbitrary Unidirectional Periodic Flow. Consider unidirectional flow of an incompressible micropolar fluid, induced by an arbitrary, sufficiently smooth force which is periodic in one of the coordinates. This generalized Kolmogorov flow

$$\psi=f(y),\ \sigma=\lambda g(y) \tag{25}$$

can also be investigated for stability. All variables are again assumed to be dimensionless, and the period of the functions f and g is taken equal to 2π (this can always be added due to the selection of length units). Since the current function was determined accurately within an arbitrary additive constant, we assume that

$$\langle f\rangle\equiv\frac{1}{2\pi}\int_0^{2\pi}f(y)\,dy=0.$$

Substituting (25) into the dimensionless equations (1), (2), we find the value of the force F and the relation between the functions f and g:

$$F=\frac{1}{R}\left(\frac{d^4f}{dy^4}+\lambda^2\frac{d^2g}{dy^2}\right),\quad l^2\frac{d^2g}{dy^2}=2\lambda g+\frac{d^2f}{dy^2}. \tag{26}$$

Thus, the periodic flow (25) is described by a single smooth periodic function, for example f(y), while the second function is determined from (26). The explicit relation between the functions f and g becomes particularly obvious after expanding them in Fourier series:

$$f=\sum_{-\infty}^{\infty}{}'\,a_k\,e^{iky},\quad g=\sum_{-\infty}^{\infty}{}'\,b_k\,e^{iky},\quad b_k=\frac{k^2}{l^2k^2+2\lambda}\,a_k, \tag{27}$$

where the prime in the summation indicates absence of the term with $k = 0$.

The stability of the flow (25), (26) is investigated by the method described in detail in the preceding sections. Following transition to the deformed coordinates, instead of (8) we obtain the equation

$$\varepsilon^4\frac{\partial}{\partial t}\Delta\psi+\varepsilon\frac{\partial(\Delta\psi,\ \psi)}{\partial(x,\ y)}=\frac{1-\varepsilon^2}{R_*}\left[\Delta^2\psi+\lambda\Delta\sigma-\left(\frac{d^4f}{dy^4}+\lambda^2\frac{d^2g}{dy^2}\right)\right], \tag{28}$$

where $\Delta = \partial^2/\partial y^2 + \varepsilon^2\partial^2/\partial x^2$. The remaining equations (9), (10) retain their previous shape.

In the vanishing approximation we find

$$\psi_0 = f(y) + \Phi_0(x, t), \tag{29a}$$

$$\sigma_0 = \lambda g(y), \tag{29b}$$

so that the solvability condition is automatically satisfied, since $\langle g \rangle = 0$ [this follows rather simply from (26)], and consequently:

$$\int_0^{2\pi} \sigma_0 dy = 2\pi\lambda \langle g \rangle = 0.$$

Within the first approximation the equations

$$\frac{\partial^4\psi_1}{\partial y^4} + \lambda\frac{\partial^2\sigma_1}{\partial y^2} = -R_* \frac{d^3f}{dy^3}\frac{\partial\Phi_0}{\partial x}, \tag{30a}$$

$$l^2\frac{\partial^2\sigma_1}{\partial y^2} - \lambda\left(2\sigma_1 + \frac{\partial^2\psi_1}{\partial y^2}\right) = -\lambda R_* j\frac{dg}{dy}\frac{\partial\Phi_0}{\partial x} \tag{30b}$$

show that the solution must be sought in the form

$$\psi_1 = -R_* f_1(y)\frac{\partial\Phi_0}{\partial x} + \Phi_1(x, t), \tag{31a}$$

$$\sigma_1 = \lambda R_* g_1(y)\frac{\partial\Phi_0}{\partial x}. \tag{31b}$$

Substitution of (31) into (30) leads to a system of two linear ordinary differential equations in the functions f_1 and g_1:

$$\frac{d^4f_1}{dy^4} - \lambda^2\frac{d^2g_1}{dy^2} = \frac{d^3f}{dy^3}, \quad -\frac{d^2f_1}{dy^2} + 2\lambda g_1 - l^2\frac{d^2g_1}{dy^2} = j\frac{dg}{dy}. \tag{32}$$

The periodicity requirement of the functions f_1 and g_1 as well as the assumption $\langle f_1 \rangle = 0$ [this restriction does not restrict generality: the general case $\langle f_1 \rangle \neq 0$ reduces to that assumed after elementary renormalization of Φ_1, as follows from (31a)] makes it possible to determine uniquely the solution of system (32). This solution can be obtained explicitly by means of Fourier series expansions:

$$f_1 = \sum_{-\infty}^{\infty}{}' a_k^{(1)} e^{iky}, \quad g_1 = \sum_{-\infty}^{\infty}{}' b_k^{(1)} e^{iky}, \tag{33}$$
$$k^2a_k^{(1)} = -\lambda^2 b_k^{(1)} - ika_k, \quad b_k^{(1)} = ik(jb_k + a_k)(l^2k^2 + 2\lambda - \lambda^2)^{-1}.$$

The solvability conditions (17) and (18) are again automatically satisfied, since all integrals in (17), (18) are trivial at the solutions (29), (31).

In the second approximation we find from (28), (9) with account of (29), (31)

$$\frac{\partial^4\psi_2}{\partial y^4} + \lambda\frac{\partial^2\sigma_2}{\partial y^2} = -R_*\frac{\partial\Phi_1}{\partial x}\frac{d^3f}{dy^3} + R_*^2\frac{\partial^2\Phi_0}{\partial x^2}\left[f_1\frac{d^3f}{dy^3} - \left(\frac{df}{dy}\right)\left(\frac{d^2f_1}{dy^2}\right)\right] + R_*^2\left(\frac{\partial\Phi_0}{\partial x}\right)^2\left(\frac{d^3f_1}{dy^3}\right), \tag{34a}$$

$$l^2\frac{\partial^2\sigma_2}{\partial y^2} - \lambda\left(2\sigma_2 + \frac{\partial^2\psi_2}{\partial y^2} + \frac{\partial^2\Phi_0}{\partial x^2}\right) = \lambda R_* j\left[R_*\frac{\partial^2\Phi_0}{\partial x^2}\left(g_1\frac{df}{dy} - f_1\frac{dg}{dy}\right) - \frac{\partial\Phi_1}{\partial x}\frac{dg}{dy} - \left(\frac{\partial\Phi_0}{\partial x}\right)^2 R_*\frac{dg_1}{dy}\right]. \tag{34b}$$

It is not difficult to find an explicit periodic solution of (34), which is quite awkward. In fact, to obtain the principal result – determination of the critical number R_* – not all the second approximation results are required. Indeed, in the second approximation the asymptotic solvability condition (10a) is

$$R_* \frac{\partial}{\partial x} \int_0^{2\pi} \left(\frac{\partial \psi_1}{\partial y} \frac{\partial^2 \psi_0}{\partial x^2} + \frac{\partial \psi_0}{\partial y} \frac{\partial^2 \psi_1}{\partial x^2} \right) dy = \frac{\partial^4}{\partial x^4} \int_0^{2\pi} \psi_0 dy + \lambda \frac{\partial^2}{\partial x^2} \int_0^{2\pi} (\sigma_2 - \sigma_0)\, dy, \tag{35}$$

so that it is necessary to calculate only the integral from σ_2 to the period. It is determined by direct integration of (34b) over the period. A number of terms, such as $\partial^2\psi_2/\partial y^2$, drop out following integration due to periodicity

$$2\langle \sigma_2 \rangle + \frac{\partial^2 \Phi_0}{\partial x^2} = R_*^2 j \frac{\partial^2 \Phi_0}{\partial x^2} \left\langle f_1 \frac{dg}{dy} - g_1 \frac{df}{dy} \right\rangle. \tag{36}$$

The remaining integrals in (35) are calculated by the previous approximations (29), (31):

$$\left\langle \frac{\partial \psi_1}{\partial y} \frac{\partial^2 \psi_0}{\partial x^2} \right\rangle = 0, \quad \left\langle \frac{\partial \psi_0}{\partial y} \frac{\partial^2 \psi_1}{\partial x^2} \right\rangle = -R_* \frac{\partial^3 \Phi_0}{\partial x^3} \left\langle \frac{df}{dy} f_1 \right\rangle.$$

$$\langle \psi_0 \rangle = \Phi_0. \tag{37}$$

Substituting (36), (37) into (35), we reach the required expression for the critical Reynolds number R_* of stability loss:

$$R_* = \sqrt{\frac{2-\lambda}{2\left\langle f \frac{df_1}{dy} \right\rangle + \lambda j \left\langle g_1 \frac{df}{dy} - f_1 \frac{dg}{dy} \right\rangle}}. \tag{38}$$

Equation (38) provides an explicit expression for the critical number of a whole class of periodic flows of a micropolar fluid. In the special case $f = \cos y$ (38) transforms to the previous result (22). The conclusions drawn in the third section concerning stability of a Kolmogorov flow are also valid for arbitrary periodic flows of shape (25). In particular, the stability of an arbitrary flow $\psi = f(y)$ is enhanced with increasing λ. In the most realistic case $\ell^2 \sim j \ll 1$, neglecting terms $O(j, \ell^2)$ and redefining the Reynolds number as was described in the third section, from (38) we obtain the expression

$$\mathrm{Re}_* = \frac{1-\lambda/2}{1-\lambda} \left(\langle f^2 \rangle \right)^{-1/2},$$

from which the consequence of enhanced stability becomes quite obvious.

NOTATION

Ω, angular velocity of microrotation; σ, nontrivial component of angular velocity of microrotation; F, external force; x, y, Cartesian coordinates; ψ, stream function; t, time; ρ, mass density; ρJ, micromoment of inertia density; μ, dynamic viscosity coefficient; k, "adhesion" coefficient; γ, rotational viscosity coefficient; U, a characteristic velocity; a, a characteristic size (flow period); R, Reynolds number; λ, ℓ, j, dimensionless parameters defined by Eq. (5); R_*, critical Reynolds number; ε, a small parameter defined by Eq. (6); Φ, stream perturbation function; A_1, a_1, constants of (16); A_2, a_2, B_2, b_2, c_2, constants of (21); H, magnetic field intensity; I, a polynomial integral; f, g, generalized Kolmogorov flow; a_k, a'_k, b_k, b'_k, Fourier coefficients; and $f_1(y)$, $g_1(y)$, functions defined in (32).

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